Channel Estimation for a Discrete-Time RAKE Receiver in a WCDMA Downlink: Algorithms and Repercussions on SINR

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Abstract—**The conventional receiver for DS-CDMA communications is the RAKE receiver which is a matched filter (MF), matched to the operations of spreading, pulse shape filtering and channel filtering. The RAKE receiver assumes a sparse/pathwise channel model so that the channel matched filtering gets done pathwise, with delay adjustment and decorrelation per path and maximum-ratio combining of path contributions at the symbol rate. Original RAKE receivers work with continuous delays, which are tracked by an Early-Late scheme. This requires signal interpolation and leads to suboptimal treatment of diffuse portions in the channel impulse response. These disadvantages can be avoided by a discrete-time RAKE, operating at a certain oversampled rate. Proper sparse modeling of the channel is an approximation problem that requires exploitation of the limited bandwidth of the pulse shape. We propose and simulate a number of sparse channel approximation algorithms along the lines of Matching Pursuit, of which the Recursive Early-Late (REL) approach appears most promising. We also analyze and simulate the effect of channel estimation on the RAKE output SINR.**

Keywords—**RAKE receiver, discrete-time RAKE, sparse channel model, channel approximation, channel estimation.**

I. Introduction

In the Wideband CDMA (WCDMA) option of the FDD mode of the 3GPP UMTS proposal for cellular wireless communications, both uplink and downlink use DS-CDMA communications. This paper focuses on the downlink, in which a set of orthogonal periodic spreading sequences are used, to take advantage of the synchronicity (between intracell users) of the downlink. To limit interference between cells though, a cell-dependent scrambling gets added which does not destroy the orthogonality between the intracell users.

The classical single-user receiver used in the forward link is the continuous-time RAKE receiver, which is a channel matched filter (MF), where the (total) channel is the convolution of the spreading sequence, the pulse-shape filter and the multipath propagation channel. The term RAKE refers to a sparse channel impulse response model in which the finite number of specular paths leads to fingers (contributions at various delays) in the channel impulse response. The propagation channel MF is a MF to a sparsified approximation $\hat{h}_{pr}(t)$ for the propagation channel $h_{pr}(t)$, in the sense that the convolution $p(t) * \hat{h}_{pr}(t)$ should approximate the convolution $p(t) * h_{pr}(t)$.

Introducing an oversampling factor so that the Nyquist criterion (corresponding to the bandwidth of the pulse-shaping filter) is satisfied, both the propagation channel MF and the pulse-shape MF can be moved after the sampling operation (if an antialiasing filter is introduced before the sampling operation), leading to a discrete-time RAKE implementation. Assuming the true propagation channel is sparse, its discrete-time representation $\hat{h}_k^{pr}$ should ideally be about equally sparse and the estimation of the path delays becomes an issue of detecting at which sampling instants a finger should be put. Another advantage of the discrete-time RAKE is that if the true channel is diffuse, a continuous-time approximation may run into the problem of concentrating many paths around the same time instant, while in the discrete-time RAKE only resolvable paths are considered.

To have a sparse channel representation, in the discrete-time RAKE we have to apply an approximation strategy in which the convolution of the sparse channel model $\hat{h}_k^{pr}$ with the sampled pulse-shape p_k approximates the sampled version of the convolution $p(t) * h_{pr}(t)$ of the true channel and the pulse-shape (Matching Pursuit techniques).

In this paper we study two Matching Pursuit techniques for sparse channel estimation. We also analyze the normalized mean square channel estimation error and its effect on the RAKE output SINR under some idealized circumstances.

II. Multiuser Downlink Data Model

Fig. 1 shows the downlink signal model in baseband. The K users are assumed to transmit linearly modulated signals over the same linear multipath channel with additive noise and intercell interference. The symbol and chip periods T and T_c are related through the spreading factor $L = T/T_c$, which is assumed here to be common for all the users. The total chip sequence b_l is the sum of the chip sequences of all the users, each one given by the product between the nth symbol of the kth user and an aperiodic spreading sequence $w_{k,l}$ which is itself the product of a periodic Walsh-Hadamard (with unit energy) spreading sequence $\boldsymbol{c}_k = [\boldsymbol{c}_{k,0}\ \boldsymbol{c}_{k,1} \cdots \boldsymbol{c}_{k,L-1}]^T$, and a base-station specific unit magnitude complex scrambling sequence s_l with variance 1, $w_{k,l} = c_{k,l\ mod\ L}\, s_l$:

$$b_l = \sum_{k=1}^{K} b_{k,l} = \sum_{k=1}^{K} a_{k,\lfloor \frac{l}{L} \rfloor} w_{k,l} \ . \qquad (1)$$

The scrambling operation is a multiplication of chip rate sequences. The spreading operation could be represented similarly, or alternatively as a filtering of an upsampled symbol sequence with the spreading sequence as impulse response, as indicated in the figure. The chip sequence b_l gets transformed

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into a continuous-time signal by filtering it with the pulse shape $p(t)$ and then passes through the multipath propagation channel $h_{pr}(t)$ to yield the received signal $y(t)$. The receiver samples M times per chip the lowpass filtered received signal.

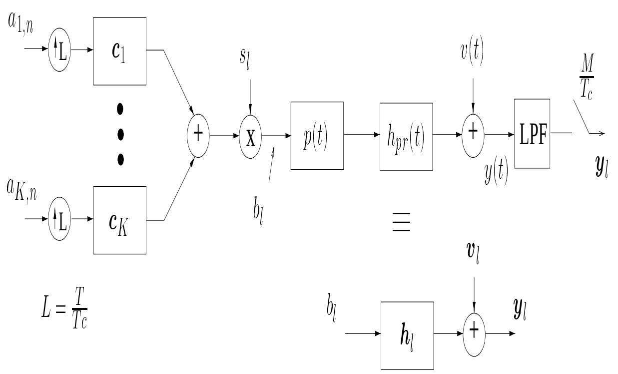

Fig. 1. Downlink signal model

Stacking the M samples per chip period in vectors, we get for the sampled received signal

$$\boldsymbol{y}_l = \sum_{k=1}^{K} \sum_{i=0}^{N-1} \boldsymbol{h}_i b_{k,l-i} + \boldsymbol{v}_l \; , \tag{2}$$

where

$$\boldsymbol{y}_l = \begin{bmatrix} y_{1,l} \\ \vdots \\ y_{M,l} \end{bmatrix}, \boldsymbol{h}_l = \begin{bmatrix} h_{1,l} \\ \vdots \\ h_{M,l} \end{bmatrix}, \boldsymbol{v}_l = \begin{bmatrix} v_{1,l} \\ \vdots \\ v_{M,l} \end{bmatrix}. \tag{3}$$

Here $\boldsymbol{h}_l$ represents the vectorized samples of the overall channel, including pulse shape, propagation channel and receiver filter. The overall channel is assumed to have a delay spread of N chips. If we model the scrambling sequence and the symbol sequences as independent i.i.d. sequences, then the chip sequence b_l is a sum of K uncorrelated white noises (chip rate i.i.d. sequences, hence stationary). The intracell contribution to $\boldsymbol{y}_l$ then is a stationary (vector) process (the continuous-time counterpart is cyclostationary with chip period). The intercell interference is a sum of contributions that are of the same form as the intracell contribution. The remaining noise is assumed to be white stationary noise. Hence the sum of intercell interference and noise, $\boldsymbol{v}_l$, is stationary.

III. RAKE RX Structure and output SINR

As shown in Fig. 2, the RAKE receiver is a channel matched filter followed by a descrambler and a correlator with the spreading code of the user of interest, which is here assumed to be user 1. If a sparse (path-wise) representation is used for the channel, then the channel matched filter leads to a RAKE structure with one finger per path. The channel matched filter is anticausal in principle, if the channel is causal. We shall assume the cascade of pulse-shape and channel to be causal so that the receiver outputs symbol estimates for the user of interest with a certain delay. The channel MF is MISO at chip rate. In Fig. 2, the operation “S/P” denotes a serial to parallel conversion which stacks the L most recent inputs into a vector. The correlator can also be viewed as a matched filter, matched to the spreading code filter, but here it is simply depicted as an inner product on a downsampled vectorized signal.

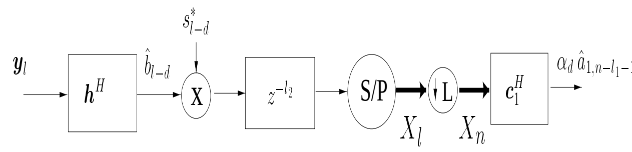

Fig. 2. The downlink receiver structure

Let’s substitute for a moment the channel MF $\boldsymbol{h}^H$ with a general linear filter $\boldsymbol{f}$. Then let $\boldsymbol{h}(z) = \sum_{l=0}^{N-1} \boldsymbol{h}_l z^{-l}$ be the $M \times 1$ FIR channel transfer function and $\boldsymbol{f}(z) = \sum_{l=0}^{P-1} \boldsymbol{f}_l z^{-l}$ the $1 \times M$ FIR filter transfer function of length P chips. Then the cascade of channel $\boldsymbol{h}$ and filter $\boldsymbol{f}$ gives $\boldsymbol{f}(z)\boldsymbol{h}(z) = \sum_{l=0}^{P+N-2} \alpha_l z^{-l} = \boldsymbol{\alpha}(z)$. In particular, for a zero-forcing (ZF) equalizer with a delay of d chips, we get $\boldsymbol{f}(z)\boldsymbol{h}(z) = z^{-d}$, but generally the symbol estimate gets produced with a certain delay of l_1+1 symbol periods where $d = l_1 L + l_2$ ($l_1 = \lfloor \frac{d}{L} \rfloor$, $l_2 = d \; mod \; L$). More precisely, the receiver outputs

$$\alpha_d \, \widehat{a}_{1,n-l_1-1} = \boldsymbol{c}_1^H \, X_n \; , \; X_n = S_{n-l_1-1}^H \mathcal{T}(\boldsymbol{f}) \boldsymbol{Y}_n \tag{4}$$

where X_n is a vector of descrambled filter outputs and $\boldsymbol{Y}_n$ is the received signal. $S_n = \text{diag}\,\{s_{n,L-1}, \ldots, s_{n,1}, s_{n,0}\}$ is a diagonal matrix of scrambling code coefficients $s_{n,l} = s_{nL+l}$, while $\mathcal{T}(\boldsymbol{f})$ is the block Toeplitz filtering matrix with $\boldsymbol{f} = [\boldsymbol{f}_0 \cdots \boldsymbol{f}_{P-1}]$ (padded with zeros) as first block row. We have for the filter-channel cascade

$$\mathcal{T}(\boldsymbol{f})\mathcal{T}(\boldsymbol{h}') = \mathcal{T}(\boldsymbol{\alpha}) = \mathcal{T}(\boldsymbol{\alpha}_d) + \mathcal{T}(\overline{\boldsymbol{\alpha}}_d) \tag{5}$$

where $\mathcal{T}(\boldsymbol{h}')$ is again a block Toeplitz filtering matrix with the zero padded $\boldsymbol{h}' = [\boldsymbol{h}_0 \cdots \boldsymbol{h}_{N-1}]$ as first block row and

$$\begin{array}{l} \boldsymbol{\alpha} = [\alpha_0 \cdots \alpha_{P+N-2}] \, , \; \boldsymbol{\alpha}_d = [0 \cdots 0 \; \alpha_d \; 0 \cdots 0] \\ \overline{\boldsymbol{\alpha}}_d = [\alpha_0 \cdots \alpha_{d-1} \; 0 \; \alpha_{d+1} \cdots \alpha_{P+N-2}] \\ \alpha_d = \boldsymbol{f}\boldsymbol{h} \; . \end{array} \tag{6}$$

In the noiseless case (and no intercell interference), the use of a ZF equalizer leads to $\overline{\boldsymbol{\alpha}}_d = [0 \cdots 0]$ and $\widehat{a}_{1,n-l_1-1} = a_{1,n-l_1-1}$ ($\alpha_d = 1$). For the RAKE receiver, $\alpha_d = ||\boldsymbol{h}||^2$, $P = N$, where $\boldsymbol{h} = [\boldsymbol{h}_{N-1}^T \cdots \boldsymbol{h}_0^T]^T$.

The analysis done in [2] shows that, due to the orthogonality of the spreading codes and to the presumed i.i.d. character of the noise, the SINR at the receiver output, Γ, is

$$\begin{array}{lcl} \Gamma_{\boldsymbol{f}} & = & \dfrac{\sigma_1^2 |\alpha_d|^2}{\boldsymbol{f} R_{VV} \boldsymbol{f}^H + \frac{\sigma_{tot}^2}{L} ||\overline{\boldsymbol{\alpha}}_d||^2} = \dfrac{\sigma_1^2 |\alpha_d|^2}{\boldsymbol{f} R_{YY} \boldsymbol{f}^H - \frac{\sigma_{tot}^2}{L} |\alpha_d|^2} \\ \Gamma_{RAKE} & = & \dfrac{\sigma_1^2 ||\boldsymbol{h}||^4}{\boldsymbol{h}^H R_{YY} \boldsymbol{h} - \frac{\sigma_{tot}^2}{L} ||\boldsymbol{h}||^4} \end{array} \tag{7}$$

where $\sigma_k^2 = \text{E}\,|a_{k,n}|^2$, $\sigma_{tot}^2 = \sum_{k=1}^K \sigma_k^2$ and $R_{YY} = R_{VV} + \frac{\sigma_{tot}^2}{L} \mathcal{T}(\boldsymbol{h}') \mathcal{T}^H(\boldsymbol{h}')$.

IV. Channel Estimation Strategies

In the classical channel estimation approach, a pilot sequence gets correlated with the received (pilot) signal. To estimate the path delays of the sparse channel, this approach looks for the positions of the maxima in this correlation (Early-Late technique). But due to oversampling and pulse-shape filtering, spurious maxima appear in the correlation, corresponding to the sidelobes of the pulse shape. The optimal approach considers a basis decomposition of the channel impulse response with the basis functions being delayed versions of the pulse shape. The basis decomposition gets estimated by an analysis-by-synthesis technique. Of course, the optimal approach leads to an exhaustive search, so suboptimal approaches are needed.

We assume that training chips are sent in every user slot during transmission. Let's define $\boldsymbol{B}_1 = \mathsf{B}_1 \otimes \boldsymbol{I}_M$ as the block Hankel matrix containing the training chip sequence of user 1 (the user of interest here); and $\boldsymbol{Y}$ is the received signal during the training sequence, vectors $\boldsymbol{g}_{M_p} = \left[g_1 g_2 \cdots g_{M_p}\right]^T$ and $\boldsymbol{\tau}_{M_p} = \left[\tau_1 \cdots \tau_{M_p}\right]$ are the (complex) path amplitudes and delays ($\tau_i \in [\tau_{min}, \tau_{max}]$, the τ_i are integers here, denoting a delay in units of sampling period); finally $\boldsymbol{P}_i(\boldsymbol{\tau}_i)$ is the matrix of the delayed pulse shape matched filters

$$\boldsymbol{P}_i(\boldsymbol{\tau}_i) = \begin{bmatrix} \vdots & & \vdots \\ p_{k-\tau_1} & \cdots & p_{k-\tau_i} \\ \vdots & & \vdots \end{bmatrix}$$

$$\boldsymbol{\tau}_i = [\tau_1 \cdots \tau_i] \qquad \boldsymbol{g}_i = [g_1 \cdots g_i]^T$$

where p_k is the oversampled root raised cosine with roll-off 0.22. The colums of $\boldsymbol{P}_i$ contain in fact samples of $p_{k-\tau}$. Then we can write the column vector containing the samples of the pulse shape - propagation channel cascade as $\boldsymbol{h} = \boldsymbol{P}_{M_p}\boldsymbol{g}_{M_p}$.

Due to the whiteness of the training chips, the least-squares fitting problem for the sparse channel parameters becomes

$$\arg \min_{\boldsymbol{\tau}_{M_p}, \boldsymbol{g}_{M_p}} ||\boldsymbol{Y} - \boldsymbol{B}_1 \boldsymbol{P}_{M_p}(\boldsymbol{\tau}_{M_p})\boldsymbol{g}_{M_p}||^2 \approx$$
$$\arg \min_{\boldsymbol{\tau}_{M_p}, \boldsymbol{g}_{M_p}} ||\widehat{\boldsymbol{h}} - \boldsymbol{P}_{M_p}(\boldsymbol{\tau}_{M_p})\boldsymbol{g}_{M_p}||^2$$

where $\widehat{\boldsymbol{h}} = \left(\boldsymbol{B}_1^H \boldsymbol{B}_1\right)^{-1} \boldsymbol{B}_1^H \boldsymbol{Y} \approx \beta^{-1}\boldsymbol{B}_1^H \boldsymbol{Y} = \sum_n B_1^*(n)\boldsymbol{y}(n)$ and β represents the training chip sequence energy. Note that the sparse channel estimation problem is separable: it becomes a two step process in which an FIR channel gets estimated in the first step and the FIR channel estimate gets approximated by a sparse model in a second step. The second step becomes Least-Squares fitting problem between the FIR estimate $\widehat{\boldsymbol{h}}$ of the overall channel and a sparse model $\boldsymbol{h} = \boldsymbol{P}_{M_p}\boldsymbol{g}_{M_p}$ of it. The sampling rate discrete-time channel impulse response can be written as $h_n = \sum_{i=1}^{M_p} g_i p_{n-\tau_i}$.

A. *Recursive Least-Squares Fitting (RLSF)*

This class of techniques minimizes with respect to the path delays and amplitudes the error between the FIR estimate of the overall channel and its theoretical expression. At each iteration (path to be added), we can reoptimize the amplitudes found in the previous steps or not (Matching Pursuit). Within this class, the best technique we can envisage (amplitude reoptimization at every position) is then:

RLSF for $i = 1, \cdots, M_p$

$$\widehat{\tau}_i = \arg \min_{\tau_i, \boldsymbol{g}_i} ||\widehat{\boldsymbol{h}} - \boldsymbol{P}_i(\widehat{\boldsymbol{\tau}}_i)\boldsymbol{g}_i||^2$$

end

B. *Recursive Early-Late (REL)*

This technique derives from the basic Early-Late approach, and corresponds to applying the Matching Pursuit technique to the convolution between the FIR estimate of the overall channel $\hat{h}_n$ and the pulse-shape matched filter (p^*_{-n}). In matrix notation (when reoptimizing jointly by LS the amplitudes):

REL $\boldsymbol{P}_0 = 0 \quad \widehat{\boldsymbol{g}}_0 = 0$

- for $i = 1, \cdots, M_p$

$$\widehat{\tau}_i = \arg \max_{\tau} |\boldsymbol{P}_1^H(\tau)\left(\widehat{\boldsymbol{h}} - \boldsymbol{P}_{i-1}(\widehat{\boldsymbol{\tau}}_{i-1})\widehat{\boldsymbol{g}}_{i-1}\right)|^2$$
$$\widehat{\boldsymbol{g}}_i = \left(\boldsymbol{P}_i^H(\widehat{\boldsymbol{\tau}}_i)\boldsymbol{P}_i(\widehat{\boldsymbol{\tau}}_i)\right)^{-1}\boldsymbol{P}_i^H(\widehat{\boldsymbol{\tau}}_i)\widehat{\boldsymbol{h}}$$

end

where multiplying with $\boldsymbol{P}_1^H(\tau)$ for all τ corresponds to convolving with p^*_{-n}. The reoptimization could be done only for the amplitude of the current iteration, in which case we have: $\widehat{g}_i = \left(\boldsymbol{P}_1^H(\widehat{\tau}_i)\boldsymbol{P}_1(\widehat{\tau}_i)\right)^{-1}\boldsymbol{P}_1^H(\widehat{\tau}_i)\widehat{\boldsymbol{h}}$. In this last case, if $f_n^0 = f_n = \widehat{h}_n * p^*_{-n}$ and $q_n = p_n * p^*_{-n}$, the loop is:

REL for $i = 1, \cdots, M_p$

$$\tau_i = \arg \max_n |f_n^{i-1}|^2$$
$$g_i = f_{\tau_i}/q_0$$
$$f_n^i = f_n - \sum_{l=1}^{i} g_l\, q_{n-\tau_l} = f_n^{i-1} - g_i\, q_{n-\tau_i}$$

end

V. Channel Estimation & SINR Degradation

We assume a specular channel model and we assume that the number of paths for the estimated channel equals the number of paths in the true channel, M_p. We shall analyze the SINR degradation due to channel estimation for a fictitious estimator that somehow knows how to put the discrete-time delays optimally. The performance results thus obtained will represent a bound for the actual performance of the algorithms considered in the previous section. We will also specialize here in a first instance the results to the case in which the true channel delays happen to fall at discrete-time instants. In that case it is immediate to define the optimal discrete-time delays.

Assume N is the channel length in chip periods and the FIR filter $\boldsymbol{f}$ is an estimate of the overall (pulse-shape plus propagation channel) channel matched filter, that is $\boldsymbol{f}^H = \hat{\boldsymbol{h}} = \boldsymbol{h} + \tilde{\boldsymbol{h}}$ ($\tilde{\boldsymbol{h}}$ is the channel estimation error - we take here $\boldsymbol{f}$ to have the same length and position as the channel matched filter). Let's define N_P as the number of training chips, $\sigma^2_{k,P}$ as the variance of training symbols for user k and $\mathsf{P} = \hat{\boldsymbol{P}}_{M_p}\left(\hat{\boldsymbol{P}}^H_{M_p}\hat{\boldsymbol{P}}_{M_p}\right)^{-1}\hat{\boldsymbol{P}}^H_{M_p}$ as the projection on the subspace spanned by the columns of $\hat{\boldsymbol{P}}^H_{M_p}$, then we can rewrite the (numerator and denominator) averaged SINR as

$$SINR = \frac{\sigma_1^2 E|\hat{\boldsymbol{h}}^H\boldsymbol{h}|^2}{E\left\{\hat{\boldsymbol{h}}^H\boldsymbol{R}_{YY}\hat{\boldsymbol{h}} - \frac{\sigma^2_{tot}}{L}|\hat{\boldsymbol{h}}^H\boldsymbol{h}|^2\right\}} \qquad (8)$$

$$= \frac{\sigma_1^2 E|\hat{\boldsymbol{h}}^H \boldsymbol{h}|^2}{\boldsymbol{h}^H \mathsf{P} \boldsymbol{R}_{YY} \mathsf{P} \boldsymbol{h} + Tr\left\{\boldsymbol{R}_{YY} \boldsymbol{C}_{\tilde{h}\tilde{h}}\right\} - \frac{\sigma_{tot}^2}{L} E|\hat{\boldsymbol{h}}^H \boldsymbol{h}|^2}$$

where E denotes expectation, $Tr\{A\}$ trace of matrix A and $\boldsymbol{C}_{\tilde{h}\tilde{h}}$ the covariance matrix of the channel estimation error, whose expression is given at the end of this section.
In the case of "no delay approximation" ($\hat{\boldsymbol{P}}_{M_p} = \boldsymbol{P}_{M_p} \Rightarrow \mathsf{P}\boldsymbol{h} = \boldsymbol{h}$) and $\frac{N_P}{N} \to \infty$ it is possible to show that the averaged SINR becomes approximatively:

$$SINR = \frac{\sigma_1^2 A}{\sigma_v^2 \|\boldsymbol{h}\|^2 + \frac{\sigma_{tot}^2}{L}\left(\|\boldsymbol{\alpha}_h\|^2 - A\right) + B} \qquad (9)$$

where $A = \|\boldsymbol{h}\|^4 + \frac{\sigma_v^2 L}{\sigma_{1,P}^2 N_P}\|\boldsymbol{h}\|^2 + \frac{\sigma_{tot,P}^2 - \sigma_{1,P}^2}{\sigma_{1,P}^2 N_P}\|\boldsymbol{\alpha}_h\|^2$,
$B = \frac{1}{\sigma_{1,P}^2 N_P}\left(L\sigma_v^4 M_p + \sigma_v^2\left(\sigma_{tot}^2 + \sigma_{tot,P}^2 - \sigma_{1,P}^2\right) Tr\{\boldsymbol{C}\}\right)$
$+ \frac{1}{\sigma_{1,P}^2 N_P}\left(\frac{\sigma_{tot}^2}{L}\left(\sigma_{tot,P}^2 - \sigma_{1,P}^2\right) Tr\{\boldsymbol{C}^2\}\right)$,
and $\boldsymbol{C} = \mathsf{P}\mathcal{T}(\boldsymbol{h})\mathcal{T}^H(\boldsymbol{h})\mathsf{P}$. The power $\sigma_{tot,P}^2$ denotes the total training symbol power, $\sigma_{tot,P}^2 = \sum_{k=1}^K \sigma_{k,P}^2$ and $\boldsymbol{\alpha}_h$ is defined as $\boldsymbol{\alpha}_h = \boldsymbol{h}^H \mathcal{T}(\boldsymbol{h}')$, so that $\alpha_N = \boldsymbol{h}^H \boldsymbol{h} = \|\boldsymbol{h}\|^2$.
The Normalized Mean Square channel estimation Error (NMSE) is given by $NMSE = \frac{Tr\{\boldsymbol{C}_{\tilde{h}\tilde{h}}\}}{\|\boldsymbol{h}\|^2}$, where

$$\boldsymbol{C}_{\tilde{h}\tilde{h}} = \frac{L\sigma_v^2}{\sigma_{1,P}^2 N_p}\mathsf{P} + \frac{\sigma_{tot,P}^2 - \sigma_{1,P}^2}{\sigma_{1,P}^2 N_p}\mathsf{P}\mathcal{T}(\boldsymbol{h})\mathcal{T}^H(\boldsymbol{h})\mathsf{P}$$

These SINR expressions for the downlink RAKE allow us to make the following observations. First, in the absence of channel estimation error, the SINR is not the same when the (intracell) interference gets replaced by white noise of the same power. Second, the effect of channel estimation error is not quite the same as when one treats the channel estimation error as an increase in additive noise.

VI. Numerical examples

To evaluate the SINR loss due to channel estimation with respect to its theoretical expression (perfect channel knowledge) of Eq. 7bis and the channel estimation error (NMSE), we performed various simulations, with different set of parameters. All the K users are considered synchronous and use the same spreading factor SF. The UMTS chip rate is assumed (3.84 Mchips/sec) and an oversampling factor of $M = 2$ is used in the simulations.

In the figures below, "true" refers to a RAKE receiver that has complete knowledge of the channel, "G estim" refers to a RAKE RX that knows the true delays but estimates the amplitudes of every path by LS fitting, "EL-LS estim" refers to a RAKE RX that estimates the propagation channel by the basic Early-late approach (puts the delays at the largest samples of $\hat{h}_k * p_{-k}^*$ and performs LS optimization of the amplitudes), "LS-OPT estim" refers to a RAKE RX that estimates the propagation channel by RLSF (section IV-A), "rec EL-LS estim" refers to a RAKE RX that estimates the propagation channel by REL (section IV-B) and "Analysis" to the theoretical performance when the true delays are integer multiples of the sampling period.

In Fig. 3 and Fig. 4, the environment is UMTS Indoor (2 paths, delay spread of about 1 μs, -10 dB avg power of the second path w.r.t. the first), spreading factor is $L = 8$ with $K = 2$ users transmitting with the same power and 20% of the slot symbols are considered training symbols.

It is remarkable to note that the basic EL approach can have better SINR performance than the true RAKE. This is indeed the case when there is one dominating path. From an SINR point of view, it is better in that case to put the taps on the main path lobe, leading to less interchip interference and hence to better SINR, than to perform channel matched filtering. But (here and in general) we can note how the NMSE performance of the REL approach is much closer to the performance of a RAKE that knows the true delay ("G estim" curve) than the basic EL performance.

References

[1] K. Insung, M.P. Fitz, and S.B. Gelfand, "Blind estimation of multipath channel parameters: a modal analysis approach," *IEEE Transactions on Communications*, vol. 47, no. 8, pp. 1140–1150, August 1999.

[2] Massimiliano Lenardi and Dirk T. M. Slock, "A RAKE Receiver with Intracell Interference Cancellation for a DS-CDMA Synchronous Downlink with Orthogonal Codes," in *Proc. VTC 2000 Spring*, Tokyo, Japan, May 2000.

[3] R. Ramésh G. Bottomley, E. Sourour and S. Chennakeshu, "Optimizing the performance of limited complexity rake receivers," in *Proc. VTC'98 Fall Conference*, 1998.

[4] Antony J. Weiss and Benjamin Friedlander, "Channel estimation for DS-CDMA downlink with aperiodic spreading codes," *IEEE Transactions on Communications*, vol. 47, no. 10, pp. 1561–1569, October 1999.

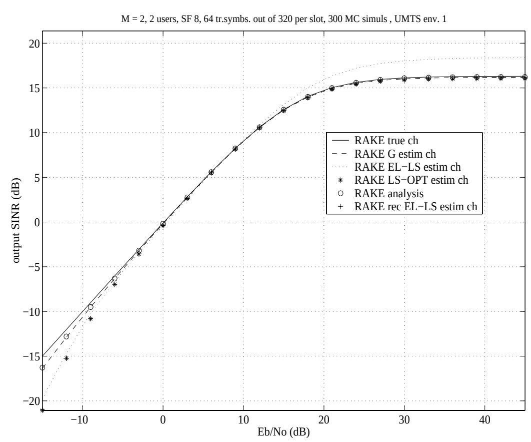

Fig. 3. Output SINR versus SNR

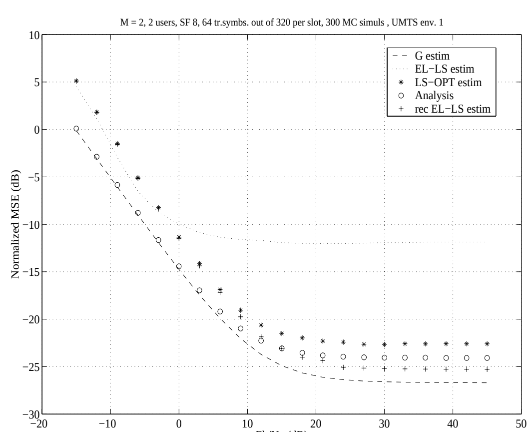

Fig. 4. NMSE versus SNR